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The Greying of Populations : Concepts and Measurement

1. Introduction

IN recent years, in response to an increasing concern in the developed world with the rapid increase in the aging of populations, there has occurred a great expansion in the literature trying to measure the aging of populations, forecast trends and analyse the socio-economic implications of aging (e.g., Kuroda and Hauser, 1981; Ogawa, 1982; Hauser, 1983). Yet, despite this great proliferation, the existing measures of aging continue to be crude. The present paper is intended to be a contribution to this problem of pure measurement.

In discussing the aging of populations we need to distinguish between the following questions :

(i) What is the age of a population ?
(ii) What is the extent of "oldness" in a population ?

While each of these questions may be interpreted in many ways and therefore have different answers, they are clearly concerned with very distinct aspects of populations. Thus the median age of a population or the average age could be thought of as answers to (i) while the percentage of population aged above 65 or above 75 are answers to (ii). In the existing literature on aging, these two questions have not always been distinguished and, for instance, the median age of a population and the percentage of population aged 65 or above (65+, henceforth) have often been thought of as alternative measures of the same larger concept (see, e.g., Hauser, 1983). In this paper we distinguish between the two and focus entirely on (ii). In the ensuing pages, a reference to the "aging of populations" is invariably a reference to the extent of "oldness" in populations.

The most commonly used index of aging simply computes the proportion of population aged 65+. For brevity, this index will be referred to as *H*. Our search for a superior measure is motivated by a dissatisfaction with *H*. We highlight its weakness by stating two axioms which any reasonable measure ought to satisfy. Then we show that both these axioms are violated by *H*. We state these axioms informally though they can easily be made mathematically precise :

Axiom C (Continuity) : An aging index should be such that a small change in the ages of the people should not cause a jump in the index.

Axiom M (Monotonicity) : If, with all other peoples' ages remaining the same, one person aged 65+ becomes older, the aging index should register an increase.

The index *H* is simply concerned with the *number* of people aged 65+. So it obviously violates *M*. To show that it violates *C*, suppose we have a population where the people are of exactly the same age and are almost 65. The index *H* of this population is clearly 0. As these people age a little and cross the 65 mark, the index will jump from 0 to 1, thereby violating Axiom *C*. Everybody becoming older by a day could change the extent of aging from 0 to the maximum possible !

Two clarifying remarks are in order. First, while we have chosen an extreme example to highlight the problem of discontinuity, a little reflection shows that the problem is quite pervasive and would crop up to a smaller extent in a wide variety of situations. Secondly, for the sake of brevity and uniformity, we adopt in this paper the language of inter-temporal changes rather than cross-sectional differences. This at times gives a false impression of "impossibility". For example, Axiom *M* seems to be applicable to a strange world where the aging process of some people can be halted. But that need not be so because at the cost of greater elaboration, *M* may be restated as follows : If there are two groups of same population size, *A* and *B*, with identical age distributions excepting for one person aged 65+ in *B* who happens to be older than his counterpart in *A* who also happens to be 65+, then the aging index of *B* should be higher than that of *A*. Similarly, later when we make observations like "suppose one person becomes younger" the reader may wish to translate this to corresponding cross-sectional comparisons of two societies.

Our aim in this paper is to suggest measures of aging which satisfy Axioms *M* and *C* and capture our underlying notion of aging more accurately. We then apply these briefly by computing the extent of aging in Japan and in India. A lot of work is this area has already been undertaken for Japan by several authors and this enables us to compare our new index with the existing ones.

In pursuing our objective of devising a suitable measure of aging, we do not have to begin from scratch. A highly developed and clearly analogous literature in economics on the problem of measuring the extent of poverty in a society (see, e.g., Sen, 1976, 1979; Takayama, 1979; Kakwani, 1980; Foster, Greer and Thorbecke, 1984; Basu, 1985) vastly simplifies the task in hand. In fact, in exploring the need for more sophisticated measures than *H*, we have already above made use of the method of Sen (1976). In what follows we draw much upon Foster, Greer and Thorbecke's family of poverty measures (see also Foster, 1984) which seems to us to have a lot to offer in developing indices of population aging.

The most popular traditional measure of the extent of poverty consists of identifying a critical level of income below which a person would be considered "poor" and then computing the proportion of population that is poor. This is known as the *head-count ratio.* The present-day method of computing aging, using 65 years as the cut-off, above which people are considered "old" and then computing the proportion of old, is an exact analogue of the head-count ratio. The head-count ratio came under criticism from Sen (1976). His paper marked the beginning of a period of search for superior poverty measures. Here, we draw considerably from this poverty measurement literature; in fact, much of the next section is an exercise in suitable adaptation of this literature to the problem of aging.

2. Alternative Measures and Axioms

This section presents the formal framework and the new measures along with their axiomatic characterizations.

A *population* will be represented here by a vector $y = (y_1 \ldots y_n)$ where n is the number of people in the population and y_i is the age of the ith oldest person, ties being broken arbitrarily. Hence, the set of all populations may be defined as :

$$X = \{y \mid y \text{ is a finite vector and } y_1 \geq y_2 \geq \ldots \geq y_n\}.$$

If θ is an index of aging then for each $y \in X$, $\theta(y)$ is a real number which indicates the extent of aging in population y. Hence, using R to denote the set of all real numbers, an aging index is a mapping :

$$\theta : X \to R. \qquad (1)$$

In much of the traditional literature a person is described as 'old' if his age is 65 or more. While we comment on this cut-off age later, for our formal analysis we do not contest this. The traditional index of aging is the proportion of old people in a population. We shall refer to this as the *head-count ratio, H.* For any $y \in X$, denoting the number of people in y by $n(y)$ and the number of people aged 65 + in y by $q(y)$, we have :

$$H(y) = \frac{q(y)}{n(y)} \tag{2}$$

This is an aging index : it is a special case of (1).

Another aging index which will form an ingredient in the measures that we develop below may be called the *age-gap index.* Given a population $y \in X$, its age-gap index, / *(y),* is given by :

$$I(y) = \sum_{i=1}^{q(y)} (y_i - 65)/q(y) \cdot 65 \tag{3}$$

This measure is created by analogy with the 'income gap' measure in the literature on poverty measurement. Note that (yi, — 65)/65 could be thought of as a measure of the extent to which person i is old. Hence the age-gap index is simply an average of the extent to which old people are old.

We have already discussed the shortcomings of the head-count measure. Among these was the fact that *H* is concerned simply with the numbers that are old. The extent of their oldness does not matter. I, on the other hand, is concerned about the extent of oldness among the old but is neutral about the numbers involved. If in one society of a 100 people only 10 are old and their age is 70, then / is 1/13. If instead all 100 people were old and were aged 70, even then I would be 1/13.

Clearly a sophisticated measure ought to take into consideration *both* the numbers of the old and the extent of their oldness. In other words, a good measure should be some kind of a combination of *H* and I. The remainder of this section is a search for such a measure

Following the poverty-measurement literature we will begin by narrowing down the aging indices possible under (1) to a more limited class comprising of a normalisation of a weighted sum of individual age-gaps of the old :

$$P(y)=A(n(y)) \sum_{i=1}^{q(y)} (y_i - 65) \; v_i(y), \tag{4}$$

where *P* is the aging index, v_i, is the weight attached to i's age-gap, *A* is the normalisation factor. Different specific measures can be derived from (4) by specifying particular forms for V{ *(y)* and *A(n(y))* or by using axioms to restrict (4). We use both methods.

In the poverty literature, $v_i(y)$ is set equal to $q(y) + 1 — i$, i.e. the rank of person i (with an older person getting a higher rank). Thus the oldest person $(i = 1)$ gets a weight of $g(y)$, the second oldest person gets $g(y) — 1$, and so on. The use of *rank* weights has some justification when talking of incomes—

as in the poverty literature. This is because incomes are usually surrogate for utility and, for utility, relative weights may make more sense than absolute ones—particularly if utility is ordinal.

In the context of aging, since absolute age has a clear meaning, it seems more sensible to use 'absolute, weights, i.e. to use the age-gaps as weights. Hence, we set, $v_i(y) = y_i - 65$, for all i, which reduces (4) to

$$P(y) = A(n(y)) \sum_{i=1}^{q(y)} (y_i - 65)^2 \qquad (5)$$

Using a suitably adapted argument from Sen (1976), consider a case where all the old people are of the same age. In that case it seems reasonable to argue that P should depend solely on the number that is old and the extent of their oldness. That is, if all the old are of the same age in y, then

$$P(y)=f(H(y),I(y)) \qquad (6)$$

Let us go along with Sen and assume (6) takes a multiplicative form, but with the reasonable additional feature of P being extra sensitive to I for higher values of I.

Axiom N (Normalisation) : If all the old are of the same age then

$$P(y) = H(y)\,(I(y))^2.$$

Axiom N applied to (5) gives us a unique and neat aging index. We denote such an index by P, and derive it in the following theorem.

Theorem I. The only aging index of the class of indices given by (5), *which satisfies axiom N, is the following :*

$$\bar{P}(y) = \frac{1}{n(y)} \sum_{i=1}^{q(y)} (y_i - 65)^2/(65)^2 \qquad (7)$$

Proof. Let $y^* \in X$ be such that all the old have the same age. Then (5) implies

$$\bar{P}(y^*) = A(n(y^*))\; q(y^*)\; (y^* - 65)^2$$

On the other hand, by axiom N and (3), we get

$$P(y^*) = H(y^*)((y^*-65)/65)^2$$

Hence

$$A(n(y^*)) = \frac{1}{(65)^2 n(y^*)}$$

Since A depends on n alone, hence for all $y \in X$,

$$A(n(y)) = \frac{1}{(65)^2 n(y)}$$

Substituting this in (5) we get (7). (QED)

Index (7) is an exact analogue of the Foster, Greer and Thorbecke (1984) measure of poverty. Among the many properties of index (7) is the fact that it satisfies the following : Suppose, of two people aged 65 +, the older person becomes older by I year and the other person becomes younger by 1 year, and everybody else's age remains unchanged. Then $\bar{P}$, will register an increase. This property is at times referred to as the *transfer axiom.* The transfer axiom seems attractive because it suggests the reasonable idea that in one year, an older person ages more than a younger one. This axiom does not however specify whether it attaches a greater weight to an older person aging (i) because the older person is old *relative* to others or (ii) because of the sheer fact of his beging advanced in age. It is our belief that most people find the transfer axiom attractive because they implicitly associate it with explanation (ii). Suppose everybody else's age remaining the same, person ; (who is above 65) becomes older by a year. The aging index will register an increase. Should the magnitude of this increase depend on what *rank* i has (in terms of age) among the old or just i s age? Most people would, we feel, choose the latter.

If we adhere to (i), we get a relative weighting scheme of the kind used by Sen. If, on the other hand, we accept (ii) — as we do here —, then we have an absolute weighting scheme as in index (7).

Index (7) captures three aspects of a population : *(a)* the number of old, *(b)* the extent of 'oldness' of the old and *(c)* the age distribution among the old. There are good reasons why all three ought to be taken into account, (a) and *(b)* have been discussed above and *(c)* is discussed in section 4. However, as argued in the concluding section there may be important practical considerations for using a simplex inder. An index which takes account of (a) and *(b)* but ignores *(c)* is a simple multiplicative combination of H and I.

$$Q(y) = H(y) \cdot I(y) \tag{8}$$

While here we state index Q directly, it is noteworthy that Q can be derived from more basic axioms. This can be achieved from a set of axioms similar to the ones specified in Basu (1985) and by applying a theorem on affine functions (Basu, 1983). Note also that Q is analogous to another measure in the Foster-Greer-Thorbecke family.

$\bar{P}$ and Q are the two measures that we wish to propose here. In the next section we actually compute these indices for India and Japan and also show how these contrast with the traditional head-count ratio.

3. Estimates

Using the population projections prepared by the United Nations (1982) to estimate population aging by the existing standard measures, Hauser (1983) concludes that the next few decades will see major increases in population aging not just in the developed or industrialised countries, but in the hitherto young populations of the developing countries as well. We would like to propose that studies such as Hauser's still underestimate the extent of aging that will occur. This is because the focus in all these studies is on the headcount ratio or some related measure. In other words, they look only at the numbers of the old. But as the numbers of the old increase, presumably their average age will increase as well, a factor which is only taken into account in the few simplistic attempts to include a measure of the 'old' old (i.e. 75+) as a proportion of the old (i.e. 65+). Once both the numbers of the old and the extent of their aging are taken into account, as is the case with our new indices (7) and (8), we will find sharper increases in aging than those suggested by the head-count ratio.

To illustrate, we compare the changes in the various indices of aging between 1980 and 2025 for the countries of Japan and India, the former one where the sudden burst in population aging has aroused considerable concern and the latter is an example of a country which has still to achieve its demographic transition but where a rise in population aging is distinctly on its way. Like Hauser, we use the population projections prepared by the United Nations (1982). The relevant population age distributions are set out in Table 1.

TABLE 1—POPULATION (IN THOUSANDS) IN VARIOUS AGE GROUPS

	Japan		*India*	
	1980	*2025*	*1980*	*2025*
Total population	116551	131451	684460	1233790
65-69	3917	6736	9450	39890
70-74	2963	6774	5951	25999
75-79	1989	6317	3136	15806
80+	1477	5817	2032	10577
65+	10346	25644	20569	92272

SOURCE : United Nations, *Demographic Indicators of Countries : Estimates and Projections as Assessed in 1980,* New York, UN, 1982.

Since the United Nations data are available by five year age groups we have assumed that all the members in each age group are aged at the mid-point of that interval. This assumption causes a difficulty with the oldest or 80+ age category. To decide on the age at which the members of this category are centred we used a simple rule of thumb. We computed the average age of those aged 80 years and above in the Indian censuses of 1971 and 1981 (Registrar General of India, 1976; Registrar General of India, 1983) as derived from the single year age distributions in these censuses. This turned out to be close to 86 years for both the 1971 and 1981 censuses and we used this parameter for all the populations in our estimates. That is, we assumed that the age of all the groups aged 80+ was 86 years.

Admittedly this is a crude method. Its defence rests on two counts. First, the number of people aged above 80 is very small in any population and a crude approximation for this group is unlikely to have a large effect on the final measure. Secondly, in most censuses there is a tendency for the ages of the old to be significantly misrecorded, so that there is little to be gained by using very refined methods for this category. For example, in the unsmoothed data of the 1971 Indian census, 19 people were stated to be aged above 160 years; and one individual claimed to be above 190, that is, he was born within a couple of years of American independence!

Once the above adjustments are made to the data, the computation of indices is straightforward. The results are given below.

TABLE 2—AGING OF POPULATION ACCORDING TO VARIOUS INDICES

	Japan			*India*		
	1980	*2025*	*% change between 1980 & 2025*	*1980*	*2025*	*% change between 1980 & 2025*
H	.6888	.1951	119.7	.0301	.0748	148.5
Q	.0116	.0315	171.5	.0034	.0089	161.8
P	.00234	.00716	206	.00062	.0017	175

SOURCE : Computed from Table 1.

Ignoring here questions_concerning the cardinality of the measures, not that both our measures, *Q* and *P*, register a sharper increase between 1980 and 2025 than does the traditional head-count ratio, *H*. This is because our measures take into account the fact that by the year 2025 there will not only be more old people, but the old people will be older. Moreover, $\bar{P}$ makes allowance for the fact that in 2025 the old will probably be more sparsely spread over the various age-groups above 65.

In comparing Japan and India one finds that both *Q* and $\bar{P}$ rise faster for Japan. This probably reflects the fact that India is still at the entrance to rapid population aging, a stage at which the average age of the old can rise relatively slowly, distributional aspects. We comment on the sociological implications of different age distributions among the old and on other aspects of aging in the next section.

4. Implications and Comments

The greying of a population can have several socioeconomic implications not all of them in the same direction. The three features of aging which are captured in our measures—the proportion that is old, the average age of the old and the distribution of the old-have different social implications.

To begin with the simplest notion of aging, that is, the numbers of people above some given cut-off point, generally sixty-five, the social and economic implications of an increase in these numbers have already been discussed in several places (see, e.g., UN, 1973; Kuroda and Hauser, 1981, Furuya and Martin, 1981; Heisel, 1984). The economic burdens imposed by a rise in the proportions of old include the increased needs for retirement benefits and specialised health services. However, at least at the earlier stages of this trend towards an older population, the economic resources to meet these new demands may be more easily available because of an unchanged aggregate dependency ratio—as the rise in the proportions of the dependent old is matched by a fall in the proportions of the dependent young with falling fertility. It is only with a stabilization of fertility and a continued increase in longevity that the problem may be more acutely felt. Indeed Preston (1984) has argued that in the United States there has occurred a diversion of resources from children to the old in response to the increasing size and influence of the old-age lobby.

Most of the above implications get accentuated in an obvious way when there is not just a rise in the proportion of the old, but a rise in the average age of the old as well. This has led to attempts to distinguish between the 'young' old and the 'old' old as in Kuroda and Hauser (1981). We feel however that both our indices provide a better measure of this concept because it uses a continuously greater weighting as a person becomes older.

The social impact of an increase in the age of a population is likely to be further complicated by the simultaneous changes that typically occur in other aspects of society as a consequence of the same modernizing forces that intiatc population aging. Of these, the two most important are probably an increase in the propensity of the young to migrate in search of a better life and a consequent or an independent breakdown of the traditional joint family (or, more specifically, multi-generation family). Both these processes have the effect of increasing the numbers of the old left to fend for themselves, at least physically and emotionally, if not economically. That such an effect is taking place is seen vividly in the rapid rise in the proportions of the old that live in single person or nuclear households in Japan (Martin and Culter, 1983), which is the first of the non-Euro-American countries to experience significant population aging. The level of single-person households is also affected by sex differentials in mortality, which are accentuated in old age in the currently low-mortality countries.

The problem of loneliness among the old indirectly highlights the importance of age distribution. The existing literature on aging is silent on how age is distributed among the old. But this distribution has implications not just for estimating the social security inputs required from the state but also for the individual psychological welfare of the aged themselves. For example, the more dispersed the age distribution of the old, the lonelier old age is likely to be and a normative index of aging should register an increase. On the other hand, with a greater clustering of the old around the average age, the old person can look forward to a greater likelihood of having his spouse and acquaintances around (Bytheway, 1970). Our aging index $\bar{\bar{P}}$ by taking account of the distribution of age among the old, makes some allowance for this loneliness factor.

5. Conclusions

There are three aspects of the aging of populations : (i) How many people are old? (ii) How old are the old? (iii) How is the age distribution among the old? An ideal measure of aging should combine all three aspects. H reflects only (i). Q reflects (i) and (ii). Only $\bar{\bar{P}}$ reflects (i), (ii) and (iii). In this sense $\bar{\bar{P}}$ is probably the best measure to use.

There is however a practical consideration which may favour Q. By squaring the age gap, $\bar{\bar{P}}$ enhances the weightage of the older people in the population. Therefore if the age reporting of the very old is poor, the accuracy of $\bar{\bar{P}}$ can be affected. On the other hand, if we cut off the age distribution at some fixed point—as we have done in this paper at age 86—then, as the population becomes older, its old age distribution improves *by definition* since the population of the old begins to cluster just below this cut-off age. This introduces a special downward bias in $\bar{\bar{P}}$. Hence Q might be the most suitable aging index given the existing data limitations. Where more detailed and reliable data are available, $\bar{\bar{P}}$ is the better measure.

Finally, a suggestion for further research. All the existing aging measures, including ours, have one important shortcoming in that, while looking into the future, they keep unchanged the 65 year cut-off point for defining the old. This would be apt if our increasing longevity was explained solely by our ability to survive up to a lower state of health conditions. In reality however one reason for our increased logevity is the fact that our health (at each corresponding age) is, on average, better. Once this factor is taken into account, it follwos that the cut off age itself should be on a sliding scale, being moved upwards as the expected longevity of a population rises.

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